THEOREM PROVERS: ONE SIZE FITS ALL?

A story about Interactive Theorem Provers and how their design impacts the ability to write verified programs efficiently for typical users.

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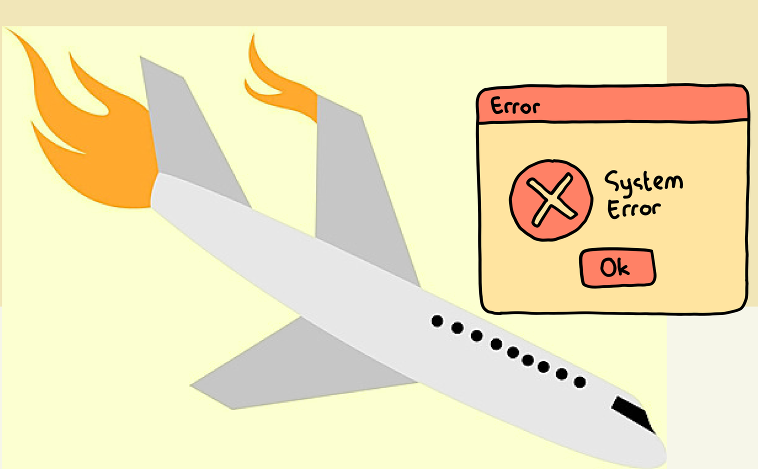
WHY COMPARE THEOREM PROVERS?

Interactive theorem provers (ITPs) help humans to formalize complex mathematics and prove the correctness of programs, potentially preventing incidents like:

- Air France Flight 447 (228 deaths)
- Therac 25 (several deaths)
- 2009 Washington Metro Collision (9 deaths, 80 severe injuries)

Users interact with ITPs in different ways depending on how they have been designed. We compare three popular systems, Coq, HOL4 and Idris2.



METHODOLOGY

As a benchmark to compare the experience, we showed the correctness of polymorphic **Insertion Sort** in each prover.

Following aspects were compared in detail:

- Installation of provers
- Proof writing
  - Defining functions/theorems
  - Interacting with the engine
  - Maintainability and recovery
- Running programs
- Community/Library support

TERMINOLOGIES

- Tactics: in Coq & HOL4, tactics are statements that specify how to manipulate the proof state to eventually reach a complete proof by decomposing the proof goal into a set of subgoals.
- Holes: in Idris2, holes allow for incomplete programs to compile so users can work on other parts of the program. Analogous to *undefined* in Haskell, or *Admitted* in Coq.
- REPL: Read-Eval-Print loop. Interactive environment that takes single user inputs, executes them and returns the result. A program written in REPL can be executed piecewise.

```
insert :: Ord A => A -> [A] -> [A]
insert i []       = []
insert i (x : xs) = if i <= x then (i : x : xs) else (x : insert i xs)

sort :: Ord A => [A] -> [A]
sort []       = []
sort (x : xs) = insert x (sort xs)
```

SCAN FOR THE PROOFS

DEFINING FUNCTIONS & THEOREMS

Coq: Functions and proofs of their properties are usually done separately. A clear correspondence can be seen between the Haskell and Coq functions.

HOL4: Like Coq, functions and proofs of their properties are done separately. As HOL4 uses Standard Meta Language (SML), the syntax is familiar to Haskell users. Unicode characters such as ∀ can be used directly in proofs — a feature Coq and Idris do not have.

Idris2: Functions, proofs and theorems are equivalent. The theorems are said to be proved if their corresponding function compiles without any hole. Consequently, functions and proofs cannot be written separately. Syntax-wise, very similar to Haskell.

INTERACTING WITH THE ENGINE

Coq: tactic-based. Each step is checked by the verified kernel as it is completed, throwing error if it is an invalid application. Coq also allows for direct manipulation of proof terms if the user desires, like Idris2.

HOL4: tactic-based, similar to Coq. Tactics manipulate proof state in SML's REPL, and are checked by the kernel for correctness in a similar manner to Coq.

Idris2: Unlike tactic-based Idris1, Idris2 takes much simpler approach, elaborating syntax directly into the core representation. This results in significant performance improvement.

MAINTAINABILITY & RECOVERY

Coq & HOL4: Due to the step-by-step nature of tactics, it is easy to jump to an earlier proof state. The interactive aspects of the provers also contribute to the ease of maintainability.

Idris2: Similar to the other two systems, the notion of helper functions (lemmas) and interactive features (case-split, add-missing) ensure good maintainability and recovery.

Search: All three systems offer proof searches, being *Search* in the Coq IDE, *DB.match* in HOL4 and *ps* in Idris2.

COMMUNITY COMPARISON

Prover	Installation Difficulty	Github Contributors	Related Research Count
Coq	Simple with binary and source	230	★ 2345
HOL4	Build from source only	65	166 (2207 for "HOL")
Idris2	Build from source only with workarounds required for Windows.	40	19 (439 for "idris")

FUTURE WORK

- Examination of the impact of proof-term vs tactic-based proving on user efficiency
- Comparison between more proof systems, such as Isabelle or Lean
- Investigation of what is achievable in terms of program performance once proofs are compiled

CONCLUSION

- Coq is good for users wanting to take advantage of a large community and more conventional proof style, while Idris2 is great for functional programmers desiring executable code. HOL4 offers a nice alternative to Coq for people desiring a different logical foundation.

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