

# Makespan Investigations of Sequential, Parallel, PO, and POCL Plans

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January 22, 2026



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**This paper investigates:**

*How do plan representations influence Makespan?*

*Makespan*: Execution time of the plan. (Here: unit costs!)

## This paper investigates:

*How do plan representations influence Makespan?*

*Makespan*: Execution time of the plan. (Here: unit costs!)

We look at, and compare, 4 typical plan structures:

- Sequential plans (totally ordered; standard)
- Parallel plans (produced by GraphPlan; used in many CSP encodings)
- Partial-order (PO) plans (not too relevant anymore, but simple!)
- PO causal link (POCL) plans (still relevant, e.g., in SAT encodings)

We challenge the common assumption:

**The last three can all achieve the same makespan.** (No...)

## The Problem: Making Breakfast

### Initial State:



Water



Coffee Beans



Bread

### Goal State:



Coffee



Toast

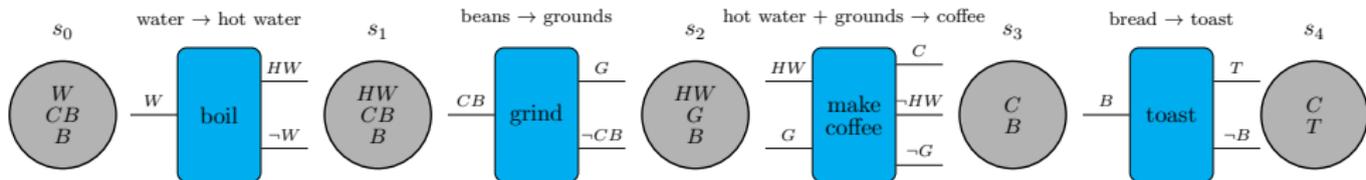
### Available Actions:

- **boil:** water → hot water
- **grind:** beans → grounds
- **make\_coffee:** hot water + grounds → coffee
- **toast:** bread → toast

**Each action takes 1 time unit.**

## Sequential Plan

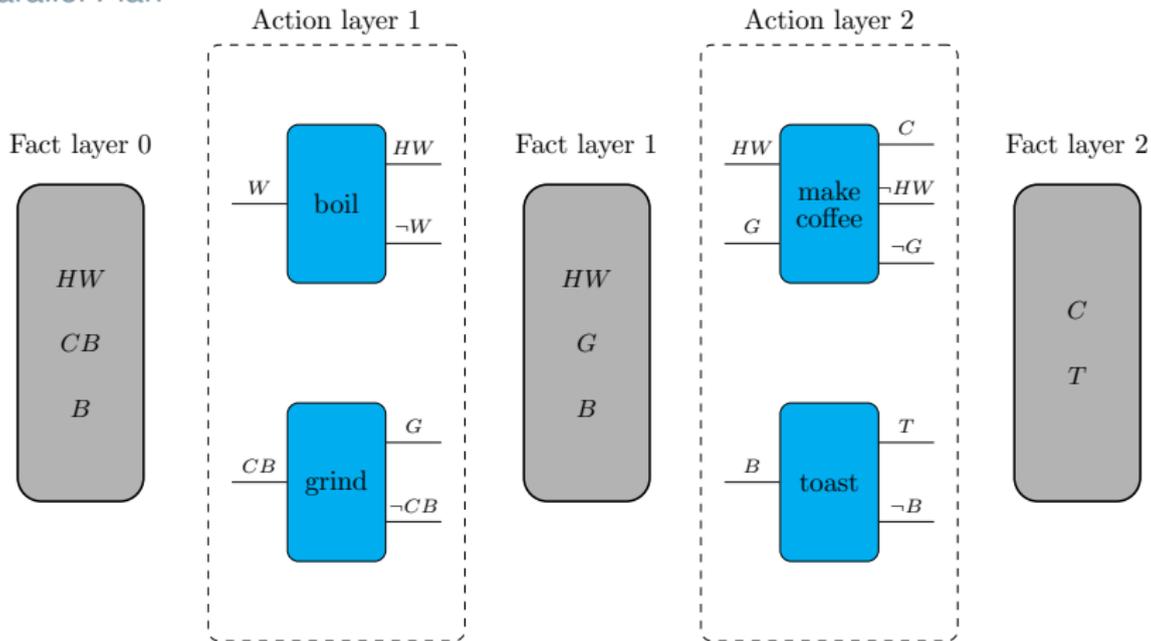
### Example plan:



Here, we have/see:

- Total-order plan has 4 actions (the boxes)
- We have 5 states (the circles), initial, 3 intermediate, and a goal state
- Each action's precondition is satisfied in its state
- Makespan is 4, and we have 1 total order.

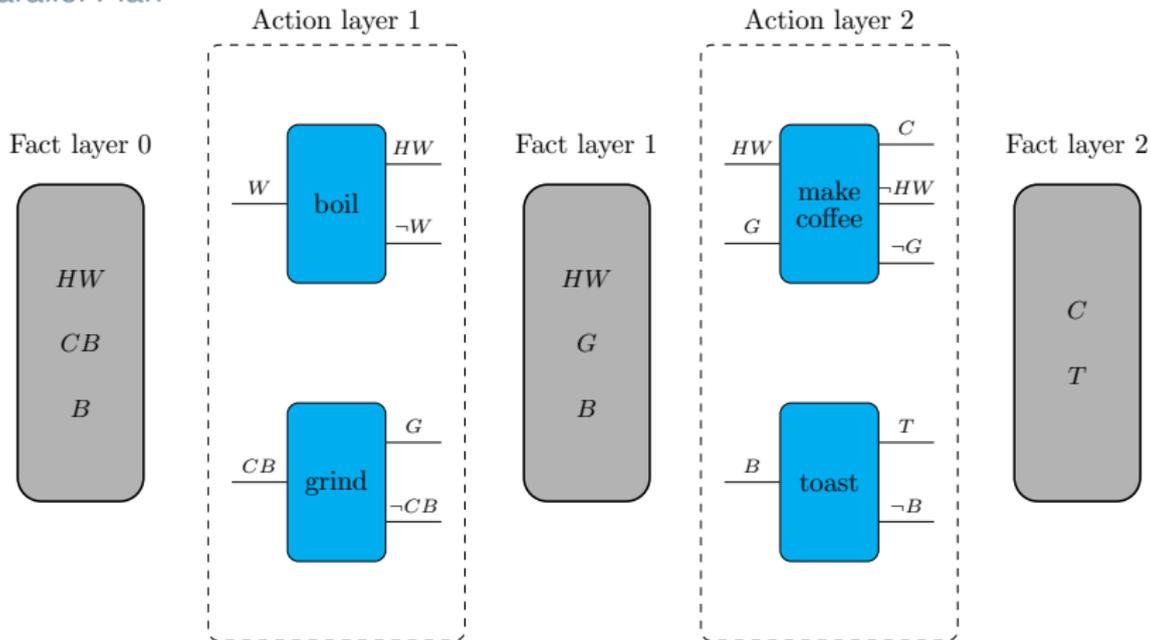
## Parallel Plan



Parallel plans are alternating fact and action layers, such that:

- All actions must be applicable in any order, and
- generate a unique successor state/fact layer

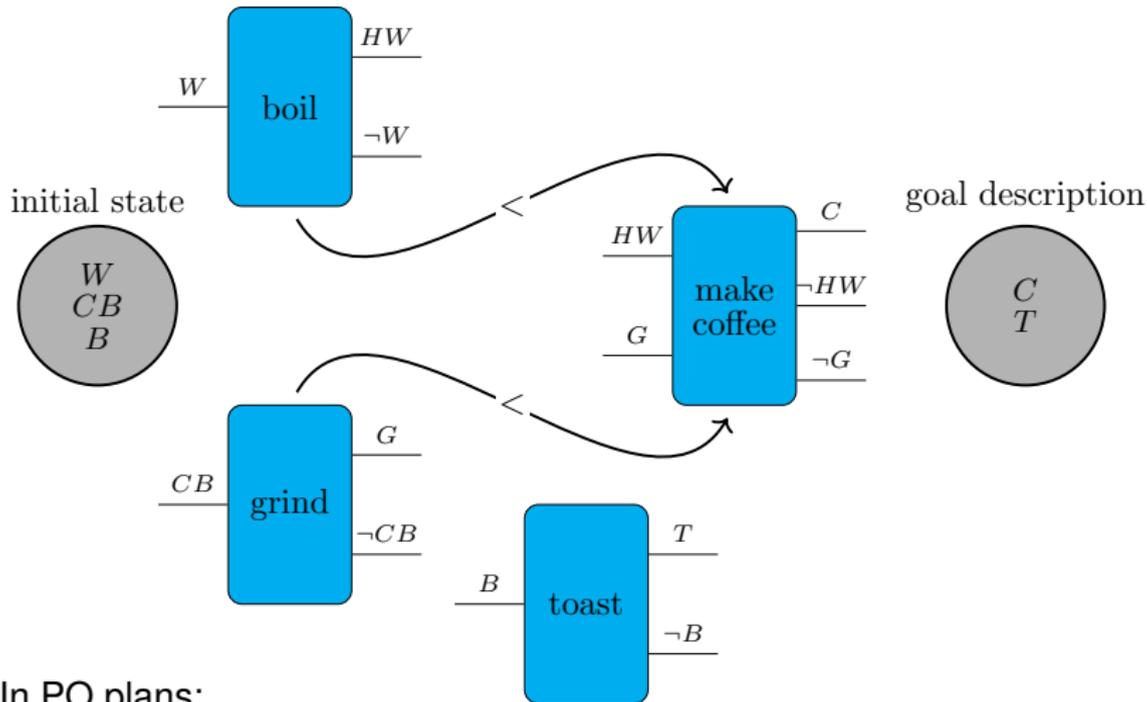
## Parallel Plan



Here, we have/see:

- Makespan is only 2!
- We have 4 total orders, but only 1 under parallelism

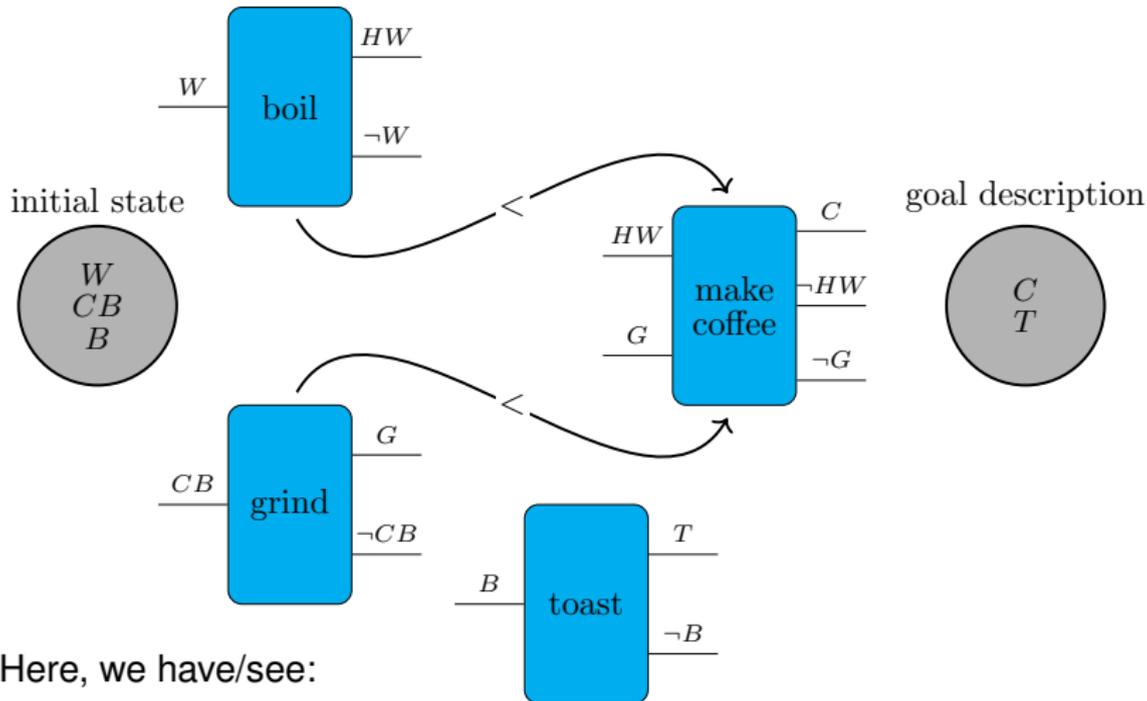
## Partial-Order (PO) Plan



In PO plans:

- Every linearization must be a sequential plan.

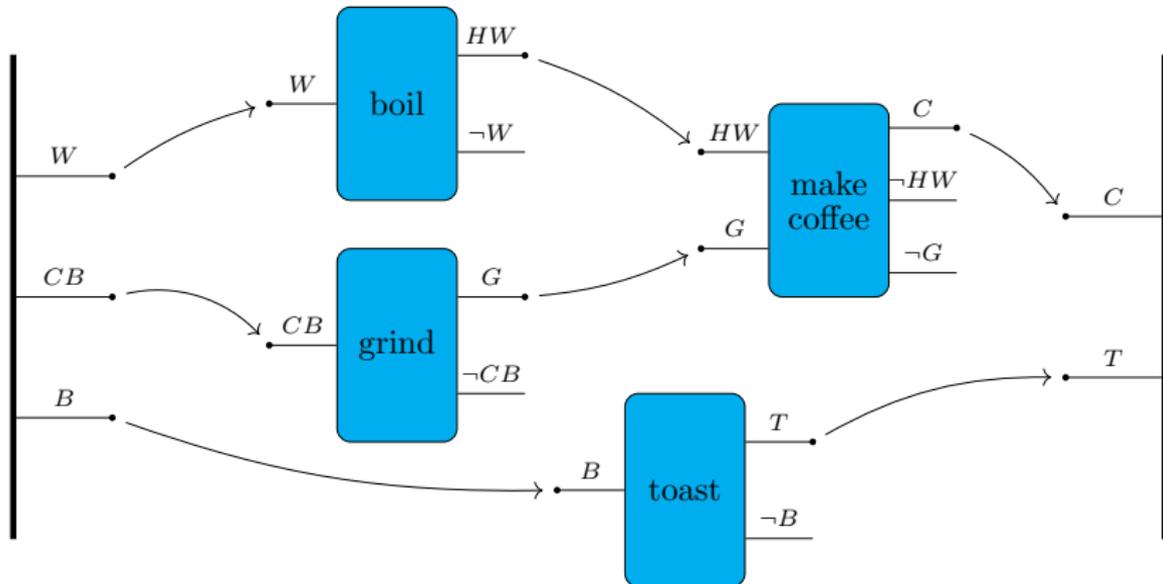
## Partial-Order (PO) Plan



Here, we have/see:

- Makespan is again 2
- We have 8 total orders, and 19 under parallelism

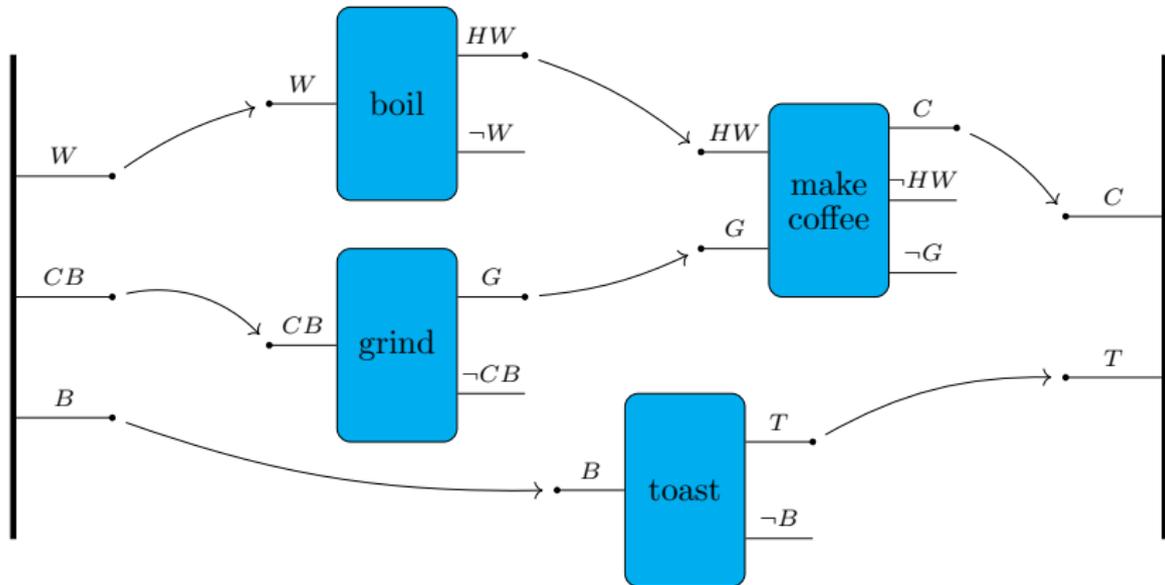
## Partial Order Causal Link (POCL) Plan



In POCL plans:

- Solution criteria connect preconditions and effects;
- they *imply* that every linearization is a sequential plan.

## Partial Order Causal Link (POCL) Plan



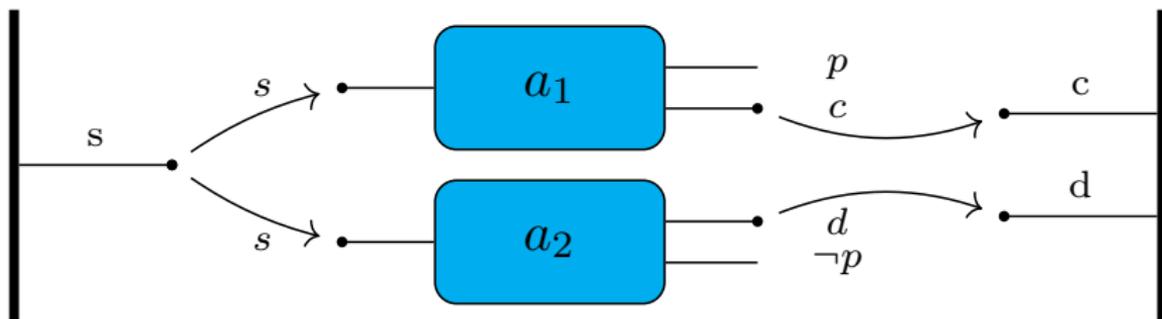
Here, we have/see:

- All properties are (coincidentally) identical to those of the PO plan
- But in general POCL plans have fewer linearizations

Parallel  $\equiv$  PO  $\equiv$  POCL?

Are parallel plans as makespan-compact as POCL plans?

Many assume so, but they are not:

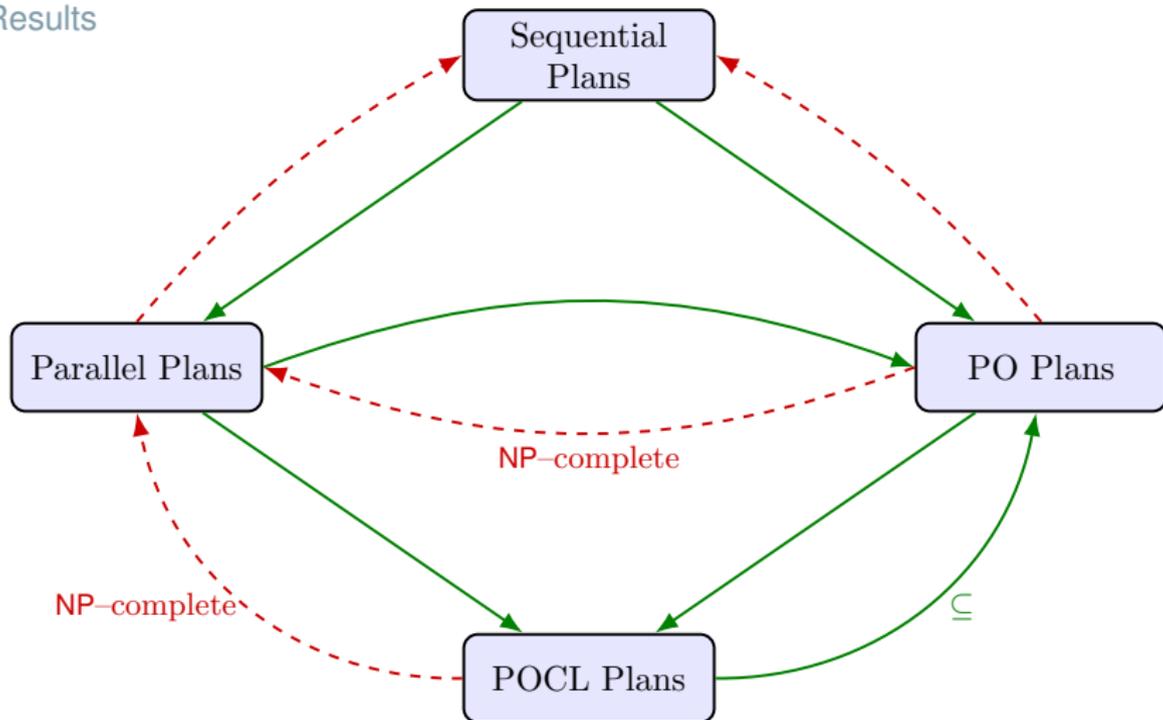


Here, we have/see:

- This POCL and PO plan has makespan 1, but:
- its parallel plan has makespan 2! (since the state is not unique.)

Can we sometimes achieve the same makespan? How to detect?

## Results



Implication:

Heuristics admissible for parallel plans are *not* admissible for POCL/PO plans!